

Correlation effects in a discrete quantum random walk

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2009 J. Phys. A: Math. Theor. 42 175304

(<http://iopscience.iop.org/1751-8121/42/17/175304>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.153

The article was downloaded on 03/06/2010 at 07:38

Please note that [terms and conditions apply](#).

Correlation effects in a discrete quantum random walk

J B Stang¹, A T Rezakhani^{1,2} and B C Sanders¹

¹ Institute for Quantum Information Science, University of Calgary, Alberta T2N 1N4, Canada

² Center for Quantum Information Science and Technology, Department of Chemistry, University of Southern California, Los Angeles, CA 90089, USA

Received 5 September 2008, in final form 10 February 2009

Published 7 April 2009

Online at stacks.iop.org/JPhysA/42/175304

Abstract

We introduce memory-dependent discrete-time quantum random walk models by adding uncorrelated memory terms and also by modifying the Hamiltonian of the walker to include couplings with memory-keeping agents. We next study numerically the correlation effects in these models. We also propose a correlation exponent as a relevant and promising tool for investigation of correlation or memory (hence non-Markovian) effects. Our analysis can easily be applied to more realistic models in which different regimes may emerge because of competition between different underlying physical mechanisms.

PACS numbers: 03.67.-a, 05.40.Fb, 03.65.Yz

(Some figures in this article are in colour only in the electronic version)

Introduction

'Random walks' (RWs) is an important and prevalent concept in various branches of science [1], in that numerous phenomena can be modeled by using the associated concepts. A classical random walk (CRW) is the dynamics of a classical (i.e., non-quantum) object—called 'particle' or 'walker' hereafter—within a stochastic environment or under some stochastic forces. The state of the walker in a CRW can be described by a local quantity, such as its 'position' (not necessarily real-space position) at each step (or 'time'), $x(t)$. A characteristic of a RW is its variance or dispersion, $\sigma^2(t) = \langle x(t)^2 \rangle - \langle x(t) \rangle^2$, where $\langle \cdot \rangle$ denotes averaging over the related probability distribution. For a simple CRW in a homogenous medium, this quantity exhibits a linear behavior with the total walk time: $\sigma^2(T) \sim T$, for sufficiently large T , which is a characteristic of a *diffusive* motion. Various generalizations of CRW can be found in the literature—for example, see [2–7] and the references therein.

Recently, there has been a great interest in the dynamics of a *quantum* random walk (QRW), a quantum object hopping (discretely or continuously) on a graph—e.g., a line—based on an intrinsically quantum-mechanical decision making in each step, e.g., by an auxiliary quantum 'coin' in a discrete QRW or by the very nature of a Hamiltonian dynamics in a continuous QRW [8–18]. Moreover, it has been shown that the language of a (continuous)

QRW, assisted with suitable Hamiltonian maps, can provide a universal framework for the studies on general qubit systems [19]. One should note that in this type of RWs an external object like a quantum coin is not necessary. In a QRW, the combined dynamics of the coin and the walker is governed by a unitary operation U_{CW} (acting on $\mathcal{H}_C \otimes \mathcal{H}_W$), which introduces quantum effects such as coherence and entanglement and results in interference between classical paths. This quantum nature is responsible for the features radically different than those of a CRW, such as: a different, spread non-Gaussian probability distribution $P(x, t)$ [12, 16], a quadratically faster spreading $\sigma^2 \sim T^2$ [12, 16, 20, 21] (the *ballistic* motion), the exponentially faster propagation between particular nodes of a specific graph [11, 13].

Our chief interest is to numerically investigate how ‘memory-effects’ or ‘correlations’ show up and play a role in the general behavior of a (discrete, coined) QRW. In a RW, the dynamics in every time is generally dictated by the history of the previous step(s) and the coin-flip(s). When the dynamics is Markovian, in principle there is no memory in the system, and the walker’s immediate future is decided only based on its present and an immediate coin-flip [1]. In other words, in a Markovian CRW, by definition, the walker does not keep any memory of its state in previous times. From a physical point of view, it seems that when the walker is interacting with a slowly responding environment—slow relative to the characteristic time of the walk—it is unlikely that the environment can feed some of the acquired (or leaked) information back to the walker, and therefore, affect its future moves. In this case, the leading effect would be a loss of memory, and accordingly, emergence of a regime in which the Markovian assumption is a valid approximation. As a result, it is expected that in this regime, there would be a negligible correlation between the configurations in distant times. In open quantum systems the analysis of non-Markovian effects is more involved than in the classical case and adding a memory kernel to the dynamical equations has various complex aspects [22–24]. Besides, unlike the classical case, in open quantum systems one cannot use the standard approaches like the Chapman–Kolmogorov equation to test the Markovian property [25], because the related joint probabilities might be not well defined from quantum-mechanical perspective. Thus, more intricate techniques are needed in order for deciding Markovianness of a quantum operation. Recently, some progress in this line has been reported in [26], where a necessary and sufficient measure for Markovianness has been suggested.

In a discrete QRW, the stepwise coin-walker dynamics, i.e., $|\Psi(t+1)\rangle_{CW} = U_{CW}|\Psi(t)\rangle_{CW}$ for $t \leq T$, may imply a Markovian characteristic for the walker’s dynamics as well. Although for the coin-walker system the dynamics is indeed Markovian (or memoryless), this is not generally the case for the walker’s state alone [27]. It has been shown that due to the quantum entanglement between the coin’s and the walker’s states, there can exist a ‘pseudo’ memory (hence non-Markovian effect) in the RW after tracing over the coin at the last step (or after a few steps). Precisely speaking, in the case of a standard discrete-time QRW with a localized initial state, the quantum probability distribution $P_{\text{quant}}(x, t)$ is related to *all* classical probabilities $\{P_{\text{class}}(x, t') : t' \leq t\}$ (with the same initial conditions). Tracing over the coin immediately after each step, and averaging over all possible measurement results, generates a CRW [28]. This is typical of systems under decoherence or interaction with an external environment. When the environment observes or measures the system, some coherence, hence dynamical information, would become inaccessible and the system tends to lose its quantum correlations. This has been anticipated as a usual route to manifest classical-like behavior in quantum systems.

In the case of a QRW, there are numerous studies specializing on how different sources of decoherence can affect a QRW and induce a transition to a CRW. For example, tracing over the coin after each step or replacing the used coin with a new one at each step (multiple

coins) and tracing over all of them after a while [28–30], random phase noise on the coin state [15, 31], unitary stochastic noise [32], periodic measurements on the coin and/or walker, or randomly broken links on the graph [33]—for a general review see [34]. In these studies, the long-time behavior of the variance σ^2 , has been adopted as an indicator to distinguish ‘classical’ and ‘quantum’ regimes of a RW [28]. This approach, although very fruitful in simple cases, is not necessarily conclusive in general cases in the sense that there are quantum diffusion models featuring sub-ballistic, the so-called ‘anomalous’ diffusion, or other types of behaviors [35–38]. This implies that, in general, a deeper characterization of different regimes in quantum systems by possibly stronger tools is needed. Moreover, there is still no clear understanding about possible roles memory, correlations or related environmental effects might play in appearance of different regimes in a general open system QRW or transitions between such regimes. A study in this line, therefore, might shed some light and bridge between seemingly different underlying notions and physical behaviors. Here we report a numerical preliminary step that may fill some blanks.

In the following, we introduce a few simple QRW models in which a memory-keeping feature is included. First we add a non-Markovian property to a QRW as an uncorrelated mixing of the states at different instants. We show how variance for these models behave versus time, signaling the inadequacy of this quantity for distinguishing different regimes. Next, we consider a more physically motivated model in which, in addition to the coin and the walker, a simple harmonic oscillator has been coupled to play as a memory-keeping agent. We define the concepts of correlation and correlation exponent as possibly useful tools for evaluating the effect of memory. The correlation exponent for our model is calculated numerically and contrasted with the exponents of a memory-dependent CRW model [6]. This analysis implies that adding memory may induce anti-correlation similar to what is seen in a self-avoiding CRW.

Uncorrelated memory dependence

We start by a brief review of the standard (memoryless, discrete) QRW [16]. This model consists of a finite one-dimensional integer lattice forming the walker’s space, $\mathcal{H}_W = \text{span}\{|x\rangle\}_{x=-L}^L$, and a chirality (or spin) degree of freedom, $\mathcal{H}_C = \text{span}\{|\pm\rangle\}$, constituting the coin space. The dynamics of $|\Psi(t)\rangle_{CW}$ is induced by the unitary operator

$$U_{CW}(p) = (P_+ \otimes S + P_- \otimes S^\dagger)(u_C(p) \otimes \mathbb{1}_W), \quad (1)$$

where P_\pm are the projection operators onto \mathcal{H}_C , $S = \sum_{x=-L}^{L-1} |x+1\rangle_W \langle x|$ is the shift operator on the graph and $u_C(p) = \begin{pmatrix} \sqrt{p} & \sqrt{1-p} \\ \sqrt{1-p} & -\sqrt{p} \end{pmatrix}$, $0 \leq p \leq 1$, is a unitary quantum coin tossing operator.

Intuitively, one might imagine that a memory-dependent model of a QRW can be built readily by considering that the state $|\Psi(t+M)\rangle_{CW}$ is obtained from some operation on a linear combination of the states in the M previous instants: $\{|\Psi(t+m)\rangle_{CW}\}_{m=0}^{M-1}$. This simple approach (without further sophisticated techniques) usually leads to a non-linear model in the following sense. Consider, for example, the following evolution:

$$|\Psi(t+2)\rangle_{CW} = \mathcal{V}_{CW}^{(1)}(t)|\Psi(t+1)\rangle_{CW} + \mathcal{V}_{CW}^{(2)}(t)|\Psi(t)\rangle_{CW}, \quad (2)$$

$\mathcal{V}^{(1)}$ and $\mathcal{V}^{(2)}$ are model-dependent dynamical operators. Normalization of $|\Psi(t+2)\rangle_{CW}$ yields

$$\begin{aligned} & \langle \Psi(t+1) | \mathcal{V}^{(1)} \mathcal{V}^{(1)\dagger} | \Psi(t+1) \rangle + \langle \Psi(t) | \mathcal{V}^{(2)} \mathcal{V}^{(2)\dagger} | \Psi(t) \rangle \\ & + 2 \text{Re}(\langle \Psi(t+1) | \mathcal{V}^{(1)} \mathcal{V}^{(2)\dagger} | \Psi(t) \rangle) = 1, \end{aligned} \quad (3)$$

which in general implies that the dynamical operators $\mathcal{V}^{(1)}$ and $\mathcal{V}^{(2)}$ would depend on the states $\Psi(t)$ and $\Psi(t+1)$. This is an unpleasant feature of this naive memory adding approach.

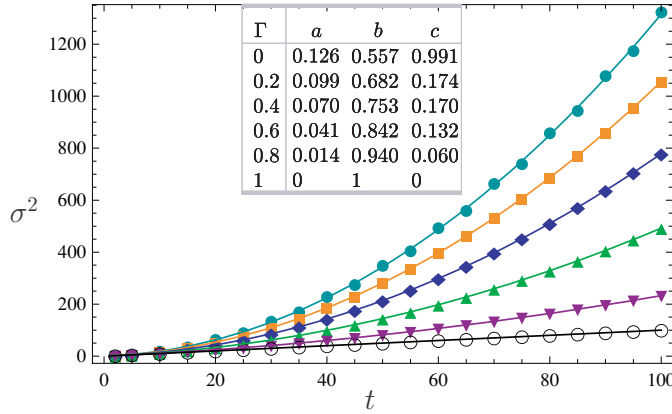


Figure 1. Variance versus time for the model in equation (4), with $M = 2, \Gamma_1 = \Gamma,$ and $\Gamma_2 = 1 - \Gamma, T = 100$ and $p = 1/2$. The inset shows the values of Γ and the parameters in the fitting $\sigma^2 \sim at^2 + bt + c$. A quadratic (linear) behavior is associated with quantum (classical) effects.

A similar issue inflicts the model $|\Psi(t + M)\rangle_{CW} = U_{CW} \sum_{m=0}^{M-1} a_m |\Psi(t + m)\rangle_{CW}$, where the coefficients a_m would depend on Ψ s. There are various ways to avoid such issues (i.e., nonlinearities). Here we consider two simple (though not necessarily physically motivated) models in which the states at different instants are mixed in an uncorrelated and random manner. A rather similar approach has already been used to investigate numerically decoherence effects on a QRW [39].

The first model is based on the density matrices in the M previous steps

$$\rho_W(t + M) = \sum_{\sigma=\pm, k=1}^M \Gamma_k A_\sigma^{(k)} \rho_W(t + M - k) A_\sigma^{(k)\dagger}. \tag{4}$$

Here, $0 \leq \Gamma_k \leq 1$, where $k \in \{1, \dots, M\}$ and $\sum_{k=1}^M \Gamma_k = 1$, is the factor determining the contribution of $\rho_W(t + M - k)$ in the dynamics of $\rho_W(t + M)$, and $A_\sigma^{(k)}$ is the Kraus operator given by $A_\sigma^{(k)} = \langle \sigma | U_{CW}^k | C \rangle$, where $|C\rangle = C_+ |+\rangle + C_- |-\rangle$ is the initial state of the coin. The non-Markovian characteristic of this model is apparent for the memory dependence of the walker’s state on its M previous instances. Another feature inferred from equation (4) is that in every step we discard the coin, tracing over it to obtain ρ_W , and use a fresh coin prepared as $|C\rangle$ for the next step (multiple coins). Indeed, including multiple coins has already been identified as a way to include previous history in a QRW [30]. For a balanced initial coin ($C_+ = -iC_- = 1/\sqrt{2}$) and a balanced coin-flip ($p = 1/2$), if $\Gamma_1 = 1$ we obtain a behavior similar to an unbiased memoryless CRW in the sense of variance (see figure 1). Degree of memory dependence of the model is adjusted by Γ_k s. For numerical simulations, in addition to the aforementioned conditions, we have taken $M = 2$ and $T = 100$, with the walker initially localized at the origin. Figure 2 depicts the peaks of the probability distribution $P(x, 100)$ for varying Γ . Note that for a range of values of Γ , the probability distribution exhibits a bimodal behavior, similar to a memoryless QRW. As Γ increases, the peaks approach each other and eventually merge into one for $\Gamma \approx 0.8$. As indicated in the inset of figure 1, for this walk a $\sigma^2 \sim a(\Gamma)t^2 + b(\Gamma)t + c(\Gamma)$ fitting can be found for different values of Γ —with $\sigma^2 \sim t$ for $\Gamma = 1$. In this respect this model shows characteristics of both QRW and CRW for different ranges of Γ .

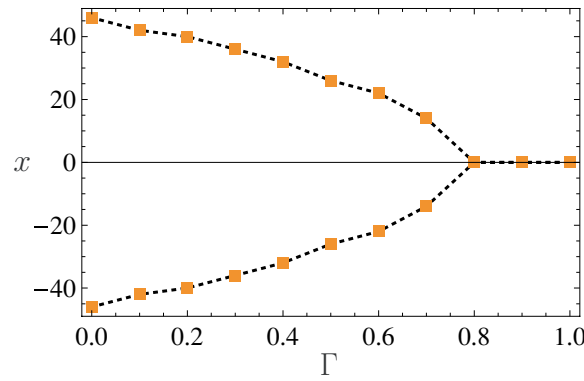


Figure 2. Positions of the peaks of the probability distribution at $t = 100$ for the evolution described in equation 4. This model exhibits bimodal (unimodal) behavior for $0 \leq \Gamma < 0.8$ ($\Gamma > 0.8$).

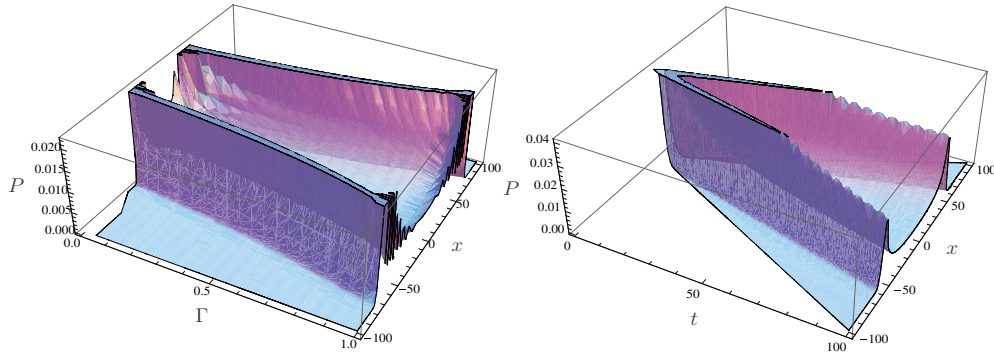


Figure 3. Left: probability distribution $P(x, 100)$ versus Γ for equation (5) and right: $P(x, t)$ for $\Gamma = 2/3$. In both of the plots we have taken $p = 1/2$.

The second model is an uncorrelated mixing of some unitary dynamics which overall constitutes a dependence on information from the M previous steps. Using M different quantum coin-walk operators, $U_{CW}(p_k) \equiv U_{CW}^{(k)}$ (for notational convenience), generated by $\{p_k\}_{k=1}^M$, the evolution is described as the following:

$$\rho_{CW}(t + M) = \sum_{k=1}^M \Gamma_k U_{CW}^{(k)} \rho_{CW}(t + M - k) U_{CW}^{(k)\dagger}. \tag{5}$$

Again, $0 \leq \Gamma_k \leq 1$ and $\sum_k \Gamma_k = 1$. The density matrix at each time $t > 1$ will involve some mixing of the different coin walks, introducing correlations. If $M = 1$ or $\Gamma_1 = 1$, this reduces to the memoryless QRW. Like equation (4), the degree of correlations can be adjusted by Γ_k . However, unlike the previous model, there is no bifurcation or transition from bimodality to unimodality in the behavior of the probability distribution versus Γ —see figure 3.

A simple harmonic oscillator model

A more physically relevant picture in which the role of correlations and memory dependence may be understood would naturally need a Hamiltonian formalism. To this end, keeping the discussion simple (for numerics), we modify the ‘Hamiltonian’ of the memoryless coined

QRW model so that a memory-keeping agent can be added. This picture can easily be generalized to more realistic cases in which the dynamics is continuous in time, and also, in principle, there is no need for an auxiliary quantum coin degree of freedom. The Hamiltonian of the memoryless QRW can be found from

$$U_{\text{CW}}(p) = e^{i\frac{\pi}{2}} e^{iZ \otimes P} e^{-i\frac{\pi}{2}(\sqrt{1-p}X + \sqrt{p}Z)} \quad (6)$$

by using the Baker–Campbell–Hausdorff formula. Now consider that the walker is coupled to a simple harmonic oscillator and a reservoir consisting of a sufficiently large number of excitation modes in the following manner:

$$H_{\text{CWOR}} = H_{\text{CW}} + H_{\text{O}} + H_{\text{R}} + H_{\text{OR}} + \lambda P(a + a^\dagger). \quad (7)$$

Although this ‘Hamiltonian’ models a crude simplification, clearly lacking various realistic characteristics, to some extent it has been inspired by a cavity QED-based proposal for a QRW [40], and serves sufficiently well for basic demonstration of our ideas. In equation (7), $H_{\text{O}} = \frac{1}{2}\omega a^\dagger a$, $H_{\text{R}} = \frac{1}{2} \sum_{k=1}^{\infty} \Omega_k c_k^\dagger c_k$, and for the coupling of the oscillator and reservoir degrees of freedom we can take, for example, $H_{\text{OR}} = g(a \sum_{k=1}^{\infty} c_k^\dagger + a^\dagger \sum_{k=1}^{\infty} c_k)$, where a is the lowering operator of the oscillator ($a^\dagger a |n\rangle_{\text{O}} = n |n\rangle_{\text{O}}$, for $0 \leq n \leq n_{\text{max}} < \infty$) and c_k is the annihilation operator of the k th excitation mode. The walker–oscillator coupling term, $H_{\text{WO}} = \lambda P(a + a^\dagger)$, implies that as the walker moves over the lattice, energy is being exchanged with the oscillator, and the oscillator gets partial information about the walker. This interaction is effectively a momentum–position coupling, which preserves the walker’s momentum. The memory dependence in this model is modulated by the coupling constant λ ($\lambda = 0$ corresponds to the memoryless QRW). The coefficient g , instead, controls the effect of the reservoir. The mediated coupling of the oscillator to the reservoir may also result in long-term correlations—this, however, needs a closer analysis which is beyond the scope of this paper. Considering that $[H_{\text{CW}}, H_{\text{WO}} + H_{\text{O}}] = 0$, we have

$$U_{\text{CWO}} = U_{\text{CW}} e^{-i(H_{\text{WO}} + H_{\text{O}})}. \quad (8)$$

To maintain tractability of the numerical simulations, here we make the following assumptions: (i) ignoring the reservoir effect, i.e., $g = 0$, (ii) taking a balanced coin-flip ($p = 1/2$) and an initially balanced coin, (iii) the walker is initially localized at the center, (iv) $\omega = 5$, (v) $T = 60$, (vi) periodic boundary condition, $S|L\rangle_{\text{W}} = |-L\rangle_{\text{W}}$, with $L = 75$, (vii) the oscillator is initially prepared at the ground state $|0\rangle_{\text{O}}$, (viii) the oscillator energy levels (or Hilbert space) are truncated at $n_{\text{max}} = 10$, and (ix) working in $0 \leq \lambda \leq 1$ interval for the coupling constant. Validity of assumption (viii) is confirmed through the simulations noting that after $T = 60$ the maximum probability for $|10\rangle_{\text{O}}$ being populated is of the order of 10^{-24} (for $\lambda = 0.1$). Moreover, we have seen that taking $L = 75$ and $T = 60$ makes the boundary effects insignificant.

Figure 4 depicts the probability distribution for the modified QRW model at $T = 60$, in which symmetry is accounted for by the symmetry of the initial conditions. The distribution with $\lambda = 0.1$ maintains a relatively similar behavior as the memoryless QRW—indicating a weak coupling regime—whilst there are non-zero probabilities everywhere on the lattice due to the λ -coupling, indicating spreading of the distribution as compared to the memoryless QRW. This can be seen in figure 5, variance $\sigma_\lambda^2(t)$ versus time t for three different values of λ , in which a fitting of the form $a(\lambda)t^2 + b(\lambda)t + c(\lambda)$ yields $a(1) \approx 2.519$, $a(0.1) \approx 0.295$ and $a(0) \approx 0.292$ —a faster spreading for the modified model, which may be understood by noting that the λ -coupling favors hopping of the walker.

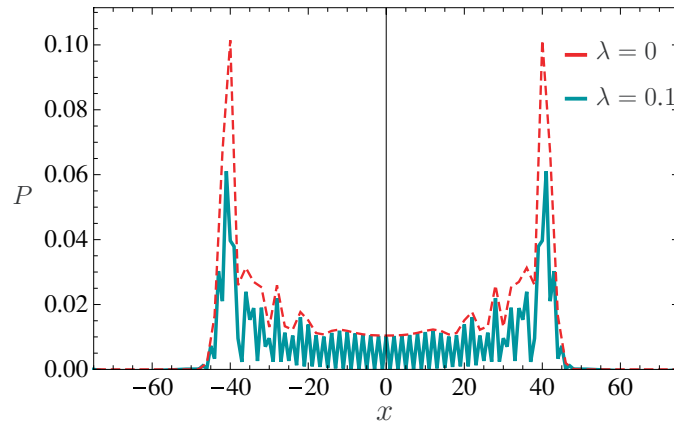


Figure 4. Probability distribution for the memory-dependent QRW described by equation (7). $\lambda = 0$ walk is only plotted at even points and for $|x| \leq 60$, as it has zero probability at odd points and for $|x| > 60$. The $\lambda = 0.1$ walk has non-zero probabilities everywhere on the lattice.

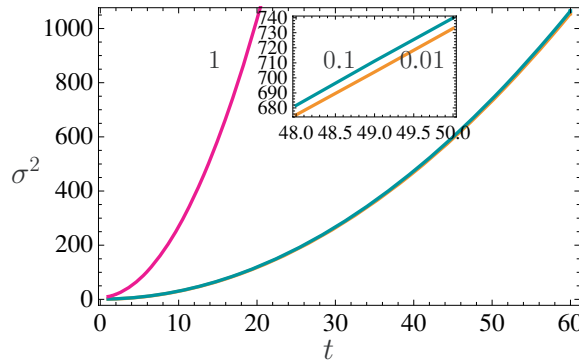


Figure 5. Variance versus time for the modified QRW for different values of λ . The inset highlights that $\sigma_{0.1}^2 > \sigma_{0.01}^2$.

Comparison of the walks

To make a meaningful comparative analysis of how memory and correlations behave in different RWs, we choose a memory-dependent CRW model proposed in [6] (for other memory-dependent CRW models see, for example, [2–4, 7]). This model is interesting in that it features various aspects of a real-world memory-keeping systems, such as ‘saturation’ of memory (i.e., finite memory capacity). Unfortunately a direct quantum-mechanical extension of the model entails nonlinearity and necessity of feedback. In this model, memory is based on the ‘information’ of the sites, which is determined by number of the times the walker visits each site as well as the times in which these visits occur. The walker hops from site i to a neighboring site j with the probability

$$p_{ij} \sim e^{u(s_j - s_i)}, \tag{9}$$

where

$$s_i(t) = \sum_m n_i(m) e^{-\kappa(t-m)} \quad (10)$$

is the remaining information at site i at time t . Here u is the density of information energy; $u > 0$ ($u < 0$) corresponds to a walker attracted to (repelled by) sites with high information content and $u = 0$ gives the memoryless CRW. The coefficient $\kappa \geq 0$ is the memory decay exponent, for our simulations fixed at $\kappa = 10^{-4}$, and $n_i(m)$ is 1 if site i was visited at time m and 0 otherwise. A saturated information amount, s_{\max} , is assumed above which the effect of information ceases to increase ($s(t) \leq s_{\max}$), which for our simulations is taken to be 13. The effect of memory on the scaling exponents of the RW, in terms of the number of sites visited and the distance the walker travels from its initial position, has been examined [6], demonstrating that for any $\kappa > 0$ and finite u this RW exhibits a scaling behavior for the variance similar to that of a memoryless CRW. As $u \rightarrow \infty$, the scaling behavior of the variance of this model changes from $\sigma^2 \sim T$ to $\sigma^2 \sim c$, where c is a constant; whereas for $u \rightarrow -\infty$, the scaling changes from $\sigma^2 \sim T$ to $\sigma^2 \sim T^2$. For our numerical analysis u remains restricted such that the variance behaves as expected for CRW. The probability distribution for this memory-dependent CRW is calculated using 10^4 independent repetitions and an averaging over the results.

A standard method to characterize short- and/or long-term memory-dependent behaviors in data analysis is through calculation of (auto-) correlation functions [41]. In the case of quantum diffusion systems, it has also been argued that the temporal scaling behavior of the correlation function can show some universal characteristic relation with the spreading and the spectrum [36–38, 42–44]. We adopt a modified definition for the correlation function as follows. From the probabilities at a fixed x^* for varying time t (up to $T \lesssim \infty$), the time-series $\{P_{x^*}(t)\}_{t=0}^T$ is generated. Next, we define

$$C_{x^*}(\tau) \equiv \sum_{t=0}^{T-\tau} P_{x^*}(t) P_{x^*}(t+\tau). \quad (11)$$

An advantage of this definition, over the one used in [36, 37, 42, 43], is that ours allows for building time-series data for a QRW in the same footing as CRW. Besides, it enables application of (classical) data analysis tools, e.g., detrended fluctuation analysis [45], for finding possible trends and fractal behaviors in a dataset. It has been shown that the correlation function exhibits a power-law (i.e., algebraic) decay as $C(\tau) \sim \tau^{-\gamma}$, where γ —the correlation exponent—is related to spectral properties of the system [36]. A small value of γ is an attribute of a walk that stays relatively localized, whereas a large γ indicates a tendency for the walker's distribution to spread with time. The correlation function has been calculated for the memory-dependent CRW (for $u = \{0, \pm 0.1\}$) and the modified QRW (for $\lambda = \{0, 0.01, 0.1\}$) at $x^* = 0$. The value of $C(\tau)$ is very small for odd values of τ . This is because the probability of the walker occupying the origin in odd ts is small. These probabilities are non-zero in the case of $\lambda > 0$ as seen earlier. In CRW and for $\lambda = 0$, this probability is zero. The even (time) points of the correlation function have been fitted to $C(\tau) \sim a + b\tau^{-\gamma}$ —figure 6—with $\gamma_{\text{QRW}} \approx 1.153$, $\gamma_{\text{SAv-CRW}} \approx 0.01$, $\gamma_{\text{CRW}} \approx 0.005$ and $\gamma_{\text{SAI-CRW}} \approx 0.003$. The behavior of γ versus λ has been plotted in figure 7. A preliminary analysis suggests that the peak in this plot may be a byproduct of the finite size of the system. A closer investigation, however, may suggest a better explanation for possible underlying reason(s). The property $\gamma_{u=0.1} < \gamma_{u=0} < \gamma_{u=-0.1}$ is in accordance with our understanding about the physical meanings of positive and negative us , i.e., self-attracting and self-avoiding, respectively. An interesting observation is that in the limit of $u \rightarrow -\infty$, SAv-CRW and QRW show similar general behaviors.

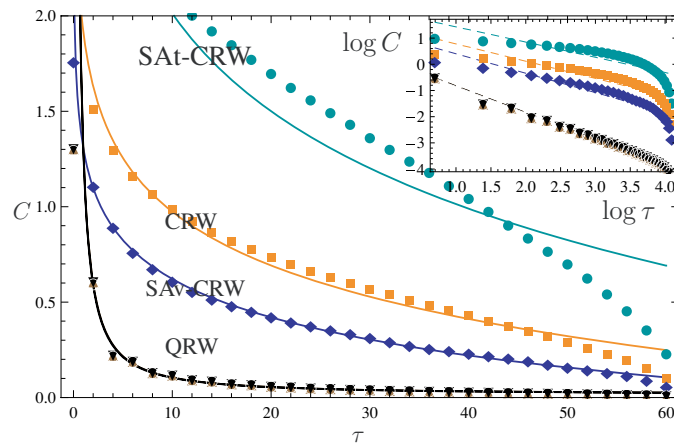


Figure 6. Autocorrelation function $C(\tau)$, for modified QRW (for $\lambda = 0, 0.01, 0.1$), self-avoiding memory-dependent CRW (SAV-CRW), memoryless CRW and self-attracting memory-dependent CRW (SAV-CRW). Closer examination reveals that, from top to bottom, the QRW plots are ordered $\lambda = 0, \lambda = 0.01$ and $\lambda = 0.1$.

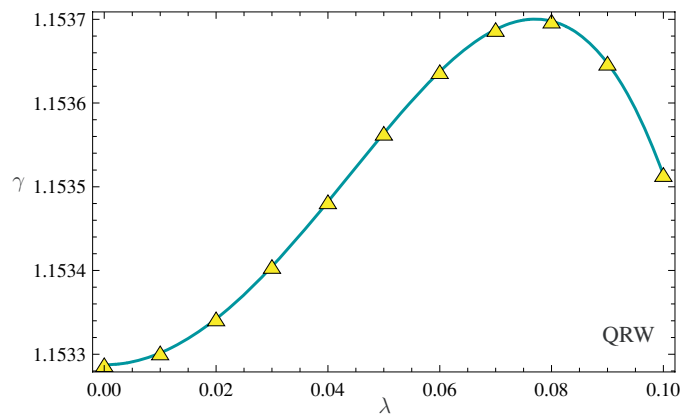


Figure 7. Plot of γ versus λ for the memory-dependent QRW. γ increases with λ , as the peak and subsequent decrease is a consequence of the finite size of our walker's space.

Summary

We have introduced simple quantum random walk models with memory-dependent features by adding a non-Markovian property to the walk as an uncorrelated mixing of the states at different instants and also by introducing a Hamiltonian picture with a memory modulating coupling. Variance versus time for the uncorrelated models has been calculated. In the Hamiltonian model, we have defined and calculated the concepts of correlation and correlation exponent as useful tools for assessing the effect of memory or correlation. Comparison with classical memoryless and memory-dependent models has indicated an anti-correlation in the quantum random walk. Variance as an indicator to distinguish between classical and quantum regimes has appeared to be a not so useful tool. We, instead, have suggested that tools

such as correlation exponents and detrended fluctuation analysis are probably more useful in characterizing different regimes of a quantum system. These studies may shed some light on how different regimes of behaviors in quantum diffusion systems emerge and how they are related to other physical characteristics of those systems. Also, these might have some implications on when a quantum random walk-based algorithm for a problem may result in a speedup in comparison to classical algorithms. A new area of application of quantum random walk ideas in which energy transfer in photosynthetic systems is considered [46] may also benefit from our study or investigations likewise.

Acknowledgments

This work was supported by CIFAR, iCORE, MITACS, NSERC and PIMS.

References

- [1] van Kampen N G 2001 *Stochastic Processes in Physics and Chemistry* (Amsterdam: North-Holland)
- Berg H C 1993 *Random Walks in Biology* (Princeton, NJ: Princeton University Press)
- Mantegna R N and Stanley H E 2000 *An Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge: Cambridge University Press)
- Motwani R and Raghavan P 1995 *Randomized Algorithms* (Cambridge: Cambridge University Press)
- [2] Hara H 1979 *Phys. Rev. B* **20** 4062
- [3] Stanley H E, Kang K, Redner S and Blumberg R L 1983 *Phys. Rev. Lett.* **51** 1223
- [4] Hilfer R 1991 *Phys. Rev. B* **44** 638
- [5] Ordemann A, Berkolaiko G, Havlin S and Bunde A 2000 *Phys. Rev. E* **61** R1005
- [6] Tan Z-J, Zou X-W, Huang S-Y, Zhang W and Jin Z-Z 2002 *Phys. Rev. E* **65** 041101
- [7] Keshet U and Hod S 2005 *Phys. Rev. E* **72** 046144
- [8] Aharonov Y, Davidovich L and Zagury N 1993 *Phys. Rev. A* **48** 1687
- [9] Meyer D A 1996 *J. Stat. Phys.* **85** 551
- [10] Watrous J 2001 *J. Comput. Syst. Sci.* **62** 376
- [11] Farhi E and Gutmann S 1998 *Phys. Rev. A* **58** 915
- [12] Nayak A and Vishwanath A 2000 arXiv:quant-ph/0010117
- [13] Childs A M *et al* 2003 *Proc. 35th ACM Symp. on Theory of Computing (STOC 2003)* (New York: ACM) p 59
- [14] Childs A M and Goldstone J 2004 *Phys. Rev. A* **70** 022314
- [15] Mackay T D, Bartlett S D, Stephenson L T and Sanders B C 2002 *J. Phys. A: Math. Gen.* **35** 2745
- [16] Kempe J 2003 *Contemp. Phys.* **44** 307
- [17] Aslangul C 2005 *J. Phys. A: Math. Gen.* **38** 1
- [18] Strauch F W 2006 *Phys. Rev. A* **74** 030301
- [19] Hines A P and Stamp P C E 2007 *Phys. Rev. A* **75** 062321
- [20] Romanelli A, Schifino A C, Sicardi, Abal G, Siri R and Donangelo R 2003 *Phys. Lett. A* **313** 325
- [21] Romanelli A, Schifino A C, Sicardi, Siri R, Abal G, Auyuanet A and Donangelo R 2004 *Physica A* **338** 395
- [22] Breuer H-P 2004 *Phys. Rev. A* **70** 012106
- Budini A A 2004 *Phys. Rev. A* **69** 042107
- Shabani A and Lidar D A 2005 *Phys. Rev. A* **71** 020101
- Breuer H-P, Gemmer J and Michel M 2006 *Phys. Rev. E* **73** 016139
- Maniscalco S and Petruccione F 2006 *Phys. Rev. A* **73** 012111
- Breuer H-P and Vacchini B 2008 *Phys. Rev. Lett.* **101** 140402
- [23] Mohseni M and Rezakhani A T 2008 arXiv:0805.3188
- [24] Breuer H-P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)
- [25] Friedrich R, Peinke J and Renner Ch 2000 *Phys. Rev. Lett.* **84** 5224
- [26] Wolf M M, Eisert J, Cubitt T S and Cirac J I 2008 *Phys. Rev. Lett.* **101** 150402
- [27] Bracken A J, Ellinas D and Tsohantjis I 2004 *J. Phys. A: Math. Gen.* **37** L91
- [28] Brun T A, Carteret H A and Ambainis A 2003 *Phys. Rev. Lett.* **91** 130602
- [29] Brun T A, Carteret H A and Ambainis A 2003 *Phys. Rev. A* **67** 052317
- [30] Flitney A P, Abbott D and Johnson N F 2004 *J. Phys. A: Math. Gen.* **37** 7581
- [31] Košik J, Bužek V and Hillery M 2006 *Phys. Rev. A* **74** 022310

- [32] Shapira D, Biham O, Bracken A J and Hackett M 2003 *Phys. Rev. A* **68** 062315
- [33] Romanelli A, Siri R, Abal G, Auyuanet A and Donangelo R 2005 *Physica A* **347** 137
- [34] Kendon V 2007 *Math. Struct. Comput. Sci.* **17** 1169
- [35] Prokof'ev N V and Stamp P C E 2006 *Phys. Rev. A* **74** 020102
- [36] Zhong J X and Mosseri R 1995 *J. Phys.: Condens. Matter* **7** 8383
- [37] Zhong J X, Diener R B, Steck D A, Oskay W H, Raizen M G, Plummer E W, Zhang Z and Niu Q 2001 *Phys. Rev. Lett.* **86** 2485
- [38] Mukhopadhyay S, Saglam M and Chatterjee 2007 *Eur. Phys. J. B* **59** 69
- [39] Maloyer O and Kendon V 2007 *New J. Phys.* **9** 87
- [40] Sanders B C, Bartlett S D, Tregenna B and Knight P L 2003 *Phys. Rev. A* **67** 042305
- [41] Honerkamp J 2002 *Statistical Physics: An Advanced Approach with Applications* (Berlin: Springer)
- [42] Ketzmerick R, Petschel G and Geisel T 1992 *Phys. Rev. Lett.* **69** 695
- [43] Ketzmerick R, Kruse K, Kraut S and Geisel T 1997 *Phys. Rev. Lett.* **79** 1959
- [44] Ribeiro P, Milman P and Mosseri R 2004 *Phys. Rev. Lett.* **93** 190503
- [45] Hu K, Ivanov P Ch, Chen Z, Carpena P and Stanley H E 2001 *Phys. Rev. E* **64** 011114
- [46] Mohseni M, Rebentrost P, Lloyd S and Aspuru-Guzik A 2008 *J. Chem. Phys.* **129** 174106